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ANALYSIS QUALIFYING EXAMINATION

January 10, 2011

SOLUTIONS

1. Let A, B be arbitrary sets of real numbers such that $a \leq 5$ for all $a \in A, b \in B$. Prove the following two statements are equivalent:

- (a) $\sup A = \inf B$
- (b) For every $\epsilon > 0$ there exist $a \in A$ and $b \in B$ such that $b - a < \epsilon$.

Solution. Assume first (a) \implies (b). Let $\epsilon = \sup A - \inf B$. Let $\epsilon > 0$. Because $\epsilon = \sup A - \inf B$, there is $a \in A$ such that $a - \epsilon < a \leq 5$. Similarly, because $\epsilon = \sup A - \inf B$, there is $b \in B$ such that $a \leq b < a + \epsilon/2$. Thus $b - a < \epsilon/2 < \epsilon$.

Conversely, assume the second condition. The fact that B is not empty and $a \leq 5$ for all $a \in A, b \in B$ implies A is bounded above and every element of B is an upper bound of A . Thus $\sup A$ exists as a real number, and $\sup A \leq 5$ for all $b \in B$. Thus $\sup A$ is a lower bound of B . It follows that $\inf B \geq \sup A$ and $\sup A \leq \inf B$. It remains to prove that $\sup A = \inf B$. Let $\epsilon > 0$ be given. By our assumption, there exist $a \in A, b \in B$ such that $b - a < \epsilon$. Thus $\inf B \leq b - a < a + \epsilon \leq \sup A + \epsilon$.

Thus $\inf B - \sup A < \epsilon < \epsilon + \epsilon = 2\epsilon$ for arbitrary $\epsilon > 0$. This implies $\inf B \leq \sup A$.

2. Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a functionally increasing and concave that

$$\lim_{x \rightarrow \infty} \int_0^x f(t) dt = \beta$$

where β is finite. Prove that $\lim_{x \rightarrow \infty} f(x) = 0$.

Solution. This is an exercise from Paul's Real Mathematical Analysis. Let $F(x) = \int_0^x f(t) dt$. Assume the conclusion to be false. Then there is a subsequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} x_n = \infty$ and still $f(x_n) \geq \delta > 0$ for all $n \in \mathbb{N}$. Assume $f(x) \geq \delta$ for all $x \in \mathbb{N}$. The case $f(x) < 0$ is similar. By uniform continuity there is $\delta > 0$ such that $|f(x) - f(y)| < \delta/2$ for $|x - y| < \delta$. It follows that $f(x) > \delta/2$ for $x \in (x_n - \delta, x_n + \delta)$ for all $n \in \mathbb{N}$. Hence $\int_{x_n - \delta}^{x_n + \delta} f(t) dt \geq \delta/2 \cdot 2\delta = \delta^2$. So, passing to a subsequence if necessary, we may assume that all the intervals $(x_n - \delta, x_n + \delta)$ are mutually disjoint. Thus $\lim_{n \rightarrow \infty} \int_{x_n - \delta}^{x_n + \delta} f(t) dt = \lim_{n \rightarrow \infty} \delta^2 = \infty$.

3. Let $\phi : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$\phi(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ x^2 & \text{if } 0 < x < 1, \\ 1 & \text{if } x \geq 1. \end{cases}$$

4. Consider the series $\sum_{n=1}^{\infty} \frac{\phi(n)}{n^2}$

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